On evaluating consensus in RANSAC surface registration

Lukáš Hruda, Jan Dvořák, Libor Váša

University of West Bohemia,
Faculty of Applied Sciences,
Department of Computer Science and Engineering
hrudalu@kiv.zcu.cz, jdvorak@kiv.zcu.cz, lvasa@kiv.zcu.cz
Two objects ($P$ and $Q$) - represent different parts of an object $O$

Both objects have some common parts (overlap)
Align $P$ and $Q$ based on their common parts

**Rigid** transformation $T$
Rigid Surface Registration

- Usually a two-step process
  1. **Global registration**
     - approximate alignment
     - fully *independent* of the initial position and orientation
  2. Local registration - refines the alignment (e.g. ICP [BM92])
Previous Work

Existing approaches

- Hough transform [Bal87, CC09]
- Phase correlation in frequency domain [BB13]
- Evolutionary algorithms [BS96, CTL04]
- Iterative - FGR [ZPK16]
- RANSAC [MPD06, AMCO08, CC09]
  - includes current state of the art Super4PCS [MAM14]
1. Numerous candidate solutions (transformations) are created
2. The candidates are evaluated - consensus evaluation
   - The best candidate(s) is(are) selected
Evaluating Consensus

- Usual approach - consensus with data
- Most common - surface overlap (Largest Common Point set - LCP) - Super4PCS
  - Counting points of $T(Q)$ close to $P$
  - Requires some distance parameter
  - Quite sensitive
  - Largest overlap $\neq$ best solution
Evaluating Consensus

- Usual approach - consensus with data
- Most common - surface overlap (Largest Common Point set - LCP) - Super4PCS
  - Counting points of $T(Q)$ close to $P$
  - Requires some distance parameter
  - Quite sensitive
  - Largest overlap $\neq$ best solution
- Instead - consensus among the candidates (mode)
Evaluating Consensus

- Not easy to evaluate
- Used previously for finding partial symmetries
  *Partial and Approximate Symmetry Detection for 3D Geometry, Mitra et al., 2006*
  - Clustering in transformation space (Mean shift)
Evaluating Consensus

- Not easy to evaluate
- Used previously for finding partial symmetries
  - *Partial and Approximate Symmetry Detection for 3D Geometry, Mitra et al., 2006*
    - Clustering in transformation space (Mean shift)
- Registration is not the same problem - clustering inappropriate
  - Outputs several clusters - only one transformation is wanted
  - Requires quite large number of candidates
  - Quite slow in general
Proposed Evaluation

- No clustering
- Instead we find the **density peak**
  - More appropriate for registration
- Proper **metric** is needed
Contribution

- Study a general RANSAC registration algorithm
  - Density peak estimation for evaluation
  - Efficient using Vantage Point Tree
- Test various transformation distance metrics
- Propose some novel improvements to one of the metrics
Model Registration Algorithm

- Points of $Q$ are paired with points of $P$ based on similar principal curvature estimates
- **Candidate transformations** are created by aligning local frames of the paired points
- The **density peak** is sought
Model Registration Algorithm

- Points of $Q$ are paired with points of $P$ based on similar principal curvature estimates
- **Candidate transformations** are created by aligning local frames of the paired points
- The **density peak** is sought
Seeking the density peak

- Density function:
  \[ \rho(T) = \sum_i K(d(T_i, T)) \]
  
- \( T_i \in \text{candidates} \)
- \( d(T_1, T_2) \) - distance function
- \( K(r) = e^{-(Dr)^2} \) - Gaussian kernel
- \( D \) - spread parameter

We do not search for the global maximum of \( \rho(T) \)
- Only maximum among candidates
Seeking the density peak

Only small values of $d(T_i, T)$ contribute significantly to $\rho(T)$

Only $T_i$ for which $d(T_i, T) < r_{\text{max}}$ are considered

Vantage Point Tree [Yia93]
- Partitions the space based on distances
- Average query complexity: $O(\log(n))$
- $d(T_1, T_2)$ must be a metric
Transformation Distance Metrics

- Decompose $T$ into rotation $R$ and translation $t$
- Weighted sum of a rotation metric and the difference of translations

$$W_1^* + W_2^* = \text{composed metric}$$
Composed metrics

\[ d(T_1, T_2) = c_R \frac{d_R(R_1, R_2)}{k_R} + c_t \frac{\|t_1 - t_2\|}{k_t} \]

- \( R_1, R_2 \) - rotation matrices, \( t_1, t_2 \) - translation vectors
- \( C_R, C_t \) - set the ratio and overall scale
- \( k_R \) - normalizes \( d_R \), so that \( d_R/k_R \in [0, 1] \)
- \( k_t \) - normalizes \( \|t_1 - t_2\| \) w.r.t. size of \( Q \)
Composed metrics - drawbacks

- Order of operations - two equivalent forms of rigid transformations
  
  - $T(x) = Rx + t$ - rotation-first form
  
  - $T(x) = R'(x + t')$ - translation-first form

- Different metric values
- Arbitrary choice
- We use rotation-first
Transformation Distance Metrics

Composed metrics - drawbacks

- Dependence on position
  - Not depending on position of $P$
  - Strongly depending on the distance of $Q$ from origin

Centered $Q$  Non-centered $Q$
Composed metrics - drawbacks

- Dependence on orientation
  - Independent of orientation of $P$ and $Q$ only if $d_R$ is **bi-invariant**
  - Bi-invariance: $d_R(R_1, R_2) = d_R(R_1 R_0, R_2 R_0) = d_R(R_0 R_1, R_0 R_2)$ for any $R_0, R_1, R_2$
Transformation Distance Metrics

Composed metrics - Used rotation metrics

1. **DEA:** 
   \[ d^{DEA}_R(R_1, R_2) = \sqrt{d(\alpha_1, \alpha_2)^2 + d(\beta_1, \beta_2)^2 + d(\gamma_1, \gamma_2)^2} \]
   \[ d(a, b) = \min\{|a - b|, 2\pi - |a - b|\} \]

2. **DQ:** 
   \[ d^{DQ}_R(R_1, R_2) = \min\{|q_1 - q_2|, |q_1 + q_2|\} \]

3. **ADPQ:** 
   \[ d^{ADPQ}_R(R_1, R_2) = \arccos(|q_1^T q_2|) \]

4. **DPQ:** 
   \[ d^{DPQ}_R(R_1, R_2) = 1 - |q_1^T q_2| \]

5. **DIM:** 
   \[ d^{DIM}_R(R_1, R_2) = \|I - R_1 R_2^T\|_F = \|R_1 - R_2\|_F \]

- **q** - unit quaternion corresponding to \( R \)
- **\( \alpha, \beta, \gamma \)** - Euler angles corresponding to \( R \)
- All except DEA are bi-invariant [Huy09]
Composed metrics - Dealing with the drawbacks

- Center $Q$ around the origin, only use bi-invariant rotation metrics
Composed metrics - Dealing with the drawbacks

- Center $Q$ around the origin, only use bi-invariant rotation metrics

OR
Transformation Distance Metrics

Composed metrics - Dealing with the drawbacks

- Center $Q$ around the origin, only use bi-invariant rotation metrics

OR

- Measure the difference of the effects on the data

= compound metric
Compound metrics - Vertex Sum of Squares (VSS)

- Squared distances of points transformed by $T_1$ from points transformed by $T_2$ [PHYH06]

$$d(T_1, T_2)^2 = \frac{c^2}{k^2} \sum_{i=1}^{n} \| T_1(q_i) - T_2(q_i) \|^2$$

- Only requires one parameter
- $c$ - the scale parameter
- $k$ - normalizes the metric w.r.t. the size and point count of $Q$

- Independent of position and orientation
- **Can be computed in $O(1)$**
- Drawback - depends on sampling density
Transformation Distance Metrics

Compound metrics - Triangle Sum of Squares (TSS)

- Solution - integrate over triangles

\[ d(T_1, T_2)^2 = \frac{c^2}{k^2} \sum_{i=1}^{\tau q} \int_{t_i} \| T_1(x) - T_2(x) \|^2 da \]

- \( t_i \) - i-th triangle
- \( c \) - the scale parameter
- \( k \) - normalizes the metric w.r.t. the size and surface area of \( Q \)
- Can be expressed in closed form
- **Still \( O(1) \) (precisely)**
Transformation Distance Metrics

Compound metric

Composed metric
14 datasets (registration problems)

Known correct transformations

Error evaluation:

$$error(T) = \frac{1}{k} \sum_{i=1}^{\tau_q} \int_{t_i} \| T(x) - T_c(x) \| \, da$$

- $T_c$ - correct transformation
- $k$ - normalizes the error w.r.t. the size and surface area of $Q$
- The triangle integrals are approximated
- Failure if $error(T) > 0.15$
Results

- 7 different metrics
  - Composed metric with all 5 rotation metrics
  - Two compound metrics: VSS, TSS
Results

- 7 different metrics
  - Composed metric with all 5 rotation metrics
  - Two compound metrics: VSS, TSS

- Optimal coefficients needed to be found
  - Exponential distribution was used
  - $c_R = 1.5 \cdot 2^{i/3}$, $c_t = 1.5 \cdot 2^{j/3}$; $i, j = 0, 1, 2, ..., 29$
  - For compound metrics $c = c_R$
Results

- 7 different metrics
  - Composed metric with all 5 rotation metrics
  - Two compound metrics: VSS, TSS
- Optimal coefficients needed to be found
  - Exponential distribution was used
  - \[ c_R = 1.5 \cdot 2^{i/3}, \quad c_t = 1.5 \cdot 2^{j/3}; \quad i, j = 0, 1, 2, \ldots, 29 \]
  - For compound metrics \( c = c_R \)
  - White: good, Black: bad, Red: estimated optimal

<table>
<thead>
<tr>
<th>DEA</th>
<th>DQ</th>
<th>ADPQ</th>
<th>DPQ</th>
<th>DIM</th>
<th>VSS</th>
<th>TSS</th>
</tr>
</thead>
</table>

Lukáš Hruda, Jan Dvořák, Libor Váša (UWB)  
On evaluating consensus in RANSAC surface registration  
July 12, 2019 25 / 30
### Results

<table>
<thead>
<tr>
<th>Dataset</th>
<th>DEA</th>
<th>DQ</th>
<th>ADPQ</th>
<th>DPQ</th>
<th>DIM</th>
<th>VSS</th>
<th>TSS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Error</td>
<td>FC</td>
<td>Error</td>
<td>FC</td>
<td>Error</td>
<td>FC</td>
<td>Error</td>
</tr>
<tr>
<td>Armadillo</td>
<td>0.0683</td>
<td>23</td>
<td>0.0479</td>
<td>0</td>
<td>0.0485</td>
<td>0</td>
<td>0.0486</td>
</tr>
<tr>
<td>Bird</td>
<td>0.0673</td>
<td>0</td>
<td>0.0672</td>
<td>0</td>
<td>0.0672</td>
<td>0</td>
<td>0.0669</td>
</tr>
<tr>
<td>Bubba</td>
<td>0.0032</td>
<td>0</td>
<td>0.0035</td>
<td>0</td>
<td>0.0039</td>
<td>0</td>
<td>0.0059</td>
</tr>
<tr>
<td>Buddha</td>
<td>0.0363</td>
<td>0</td>
<td>0.0255</td>
<td>0</td>
<td>0.0252</td>
<td>0</td>
<td>0.0246</td>
</tr>
<tr>
<td>Coati</td>
<td>0.0283</td>
<td>0</td>
<td>0.0211</td>
<td>0</td>
<td>0.0208</td>
<td>0</td>
<td>0.0212</td>
</tr>
<tr>
<td>Dragon</td>
<td>0.0258</td>
<td>0</td>
<td>0.0222</td>
<td>0</td>
<td>0.0221</td>
<td>0</td>
<td>0.0233</td>
</tr>
<tr>
<td>Eggs</td>
<td>0.0758</td>
<td>98</td>
<td>0.0852</td>
<td>120</td>
<td>0.0868</td>
<td>123</td>
<td>0.0967</td>
</tr>
<tr>
<td>Head</td>
<td>0.0079</td>
<td>0</td>
<td>0.0081</td>
<td>0</td>
<td>0.0082</td>
<td>0</td>
<td>0.0089</td>
</tr>
<tr>
<td>Hippo</td>
<td>0.0760</td>
<td>32</td>
<td>0.0560</td>
<td>0</td>
<td>0.0558</td>
<td>0</td>
<td>0.0566</td>
</tr>
<tr>
<td>Kachel</td>
<td>0.0187</td>
<td>0</td>
<td>0.0152</td>
<td>0</td>
<td>0.0156</td>
<td>0</td>
<td>0.0168</td>
</tr>
<tr>
<td>Oscar</td>
<td>0.0044</td>
<td>0</td>
<td>0.0039</td>
<td>0</td>
<td>0.0040</td>
<td>0</td>
<td>0.0048</td>
</tr>
<tr>
<td>Suzanne</td>
<td>0.0139</td>
<td>0</td>
<td>0.0133</td>
<td>0</td>
<td>0.0139</td>
<td>0</td>
<td>0.0144</td>
</tr>
<tr>
<td>Teeth</td>
<td>0.0160</td>
<td>0</td>
<td>0.0137</td>
<td>0</td>
<td>0.0135</td>
<td>0</td>
<td>0.0148</td>
</tr>
<tr>
<td>Testbody</td>
<td>0.0189</td>
<td>0</td>
<td>0.0175</td>
<td>0</td>
<td>0.0172</td>
<td>0</td>
<td>0.0197</td>
</tr>
<tr>
<td>Total</td>
<td>0.0329</td>
<td>153</td>
<td>0.0286</td>
<td>120</td>
<td>0.0288</td>
<td>123</td>
<td>0.0302</td>
</tr>
</tbody>
</table>
Results of LCP

- LCP used instead of the density peak estimation, fail count measured
- $\delta'$ is relative to the size of $Q$

<table>
<thead>
<tr>
<th>Dataset</th>
<th>$\delta'$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0025</td>
</tr>
<tr>
<td>Arm</td>
<td>619</td>
</tr>
<tr>
<td>Bir</td>
<td>887</td>
</tr>
<tr>
<td>Bub</td>
<td>0</td>
</tr>
<tr>
<td>Bud</td>
<td>112</td>
</tr>
<tr>
<td>Coa</td>
<td>59</td>
</tr>
<tr>
<td>Dra</td>
<td>330</td>
</tr>
<tr>
<td>Egg</td>
<td>215</td>
</tr>
<tr>
<td>Hea</td>
<td>0</td>
</tr>
<tr>
<td>Hip</td>
<td>708</td>
</tr>
<tr>
<td>Kac</td>
<td>368</td>
</tr>
<tr>
<td>Osc</td>
<td>0</td>
</tr>
<tr>
<td>Suz</td>
<td>90</td>
</tr>
<tr>
<td>Tee</td>
<td>0</td>
</tr>
<tr>
<td>Tes</td>
<td>375</td>
</tr>
<tr>
<td>Total</td>
<td>3763</td>
</tr>
</tbody>
</table>

Lukáš Hruda, Jan Dvořák, Libor Váša (UWB)  
On evaluating consensus in RANSAC surface registration  
July 12, 2019 27 / 30
Conclusion

- Consensus evaluation by **density peak estimation**
  - Can be made **efficient** using Vantage Point Tree
  - Times comparable to Super4PCS

- Different transformation distance metrics were compared
  - All seem to be usable
  - Composed metrics have fundamental drawbacks
  - Compound metrics have no drawbacks and perform as well or even better

- There is no reason to prefer composed over compound metrics

- The proposed TSS metric performs best

- Other possible applications
  - Comparing transformations only makes sense when related to input data
  - Compound metrics do that in a systematic way
Conclusion

- Consensus evaluation by **density peak estimation**
  - Can be made **efficient** using Vantage Point Tree
  - Times comparable to Super4PCS
- It is more **reliable** and **stable** than LCP
- Different transformation distance metrics were compared
  - All seem to be usable
  - Composed metrics have fundamental drawbacks
  - Compound metrics have no drawbacks and perform as well or even better
    \[ \Rightarrow \] There is no reason to prefer composed over compound metrics
Conclusion

- Consensus evaluation by **density peak estimation**
  - Can be made **efficient** using Vantage Point Tree
  - Times comparable to Super4PCS
- It is more **reliable** and **stable** than LCP
- Different transformation distance metrics were compared
  - All seem to be usable
  - Composed metrics have fundamental drawbacks
  - Compound metrics have no drawbacks and perform as well or even better
    - There is no reason to prefer composed over compound metrics
  - The proposed TSS metric performs best
- Other possible applications
  - Comparing transformations **only** makes sense when related to input data
  - Compound metrics do that in a systematic way
References


Thank you

Feel free to download the reference implementation
http://meshcompression.org/sgp2019