On evaluating consensus in RANSAC surface registration

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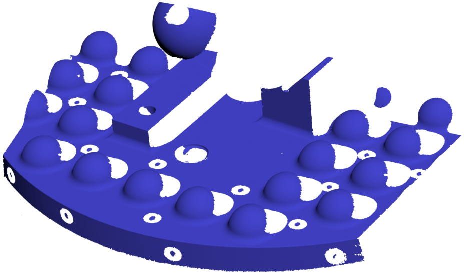
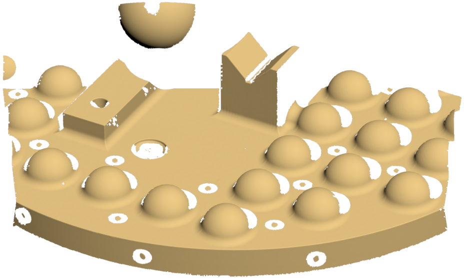
Rigid Surface Registration

- Two objects (P and Q) - represent different parts of an object O
- Both objects have some common parts (overlap)



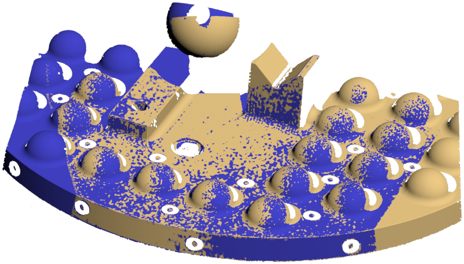
P



Q

Rigid Surface Registration

- Align P and Q based on their common parts
- **Rigid** transformation T



$P \cup T(Q)$

Rigid Surface Registration

- Usually a two-step process
  1. **Global registration**
     - approximate alignment
     - fully **independent** of the **initial position and orientation**
  2. Local registration - refines the alignment (e.g. ICP [BM92])

Previous Work

Existing approaches

- Hough transform [Bal87, CC09]
- Phase correlation in frequency domain [BB13]
- Evolutionary algorithms [BS96, CTL04]
- Iterative - FGR [ZPK16]
- RANSAC [MPD06, AMCO08, CC09]
    - includes current state of the art Super4PCS [MAM14]

RANSAC Approach

1. Numerous candidate solutions (transformations) are created
2. The candidates are evaluated - **consensus** evaluation
   - The best candidate(s) is(are) selected

Evaluating Consensus

- Usual approach - consensus with data
- Most common - surface overlap (Largest Common Point set - LCP) - Super4PCS
  - Counting points of $T(Q)$ **close** to P
  - Requires some **distance parameter**
  - Quite sensitive
  - Largest overlap $\neq$ best solution

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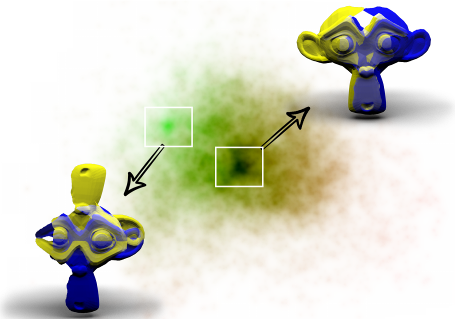
  - Largest overlap $\neq$ best solution
- Instead - consensus **among** the candidates (mode)



Evaluating Consensus

- Not easy to evaluate
- Used previously for finding partial symmetries
  Partial and Approximate Symmetry Detection for 3D Geometry, Mitra et al., 2006
    - Clustering in transformation space (Mean shift)

Evaluating Consensus

- Not easy to evaluate
- Used previously for finding partial symmetries
  Partial and Approximate Symmetry Detection for 3D Geometry, Mitra et al., 2006
  - Clustering in transformation space (Mean shift)
- Registration is not the same problem - clustering inappropriate
  - Outputs several clusters - only one transformation is wanted
  - Requires quite large number of candidates
  - Quite slow in general

Proposed Evaluation

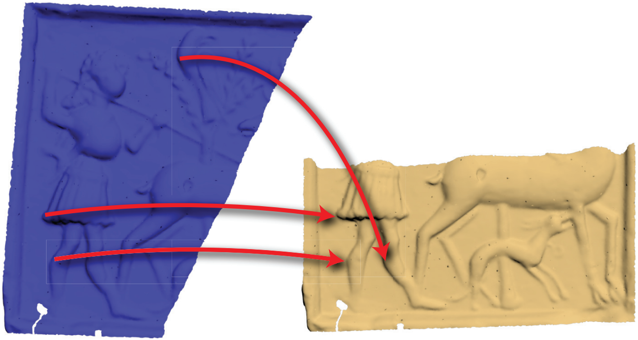
- No clustering
- Instead we find the **density peak**
  - More appropriate for registration
- Proper **metric** is needed

Contribution

- Study a general RANSAC registration algorithm
  - Density peak estimation for evaluation
  - Efficient using Vantage Point Tree
- Test various transformation distance metrics
- Propose some novel improvements to one of the metrics

Model Registration Algorithm

- Points of Q are paired with points of P based on similar principal curvature estimates
- **Candidate transformations** are created by aligning local frames of the paired points
- The **density peak** is sought



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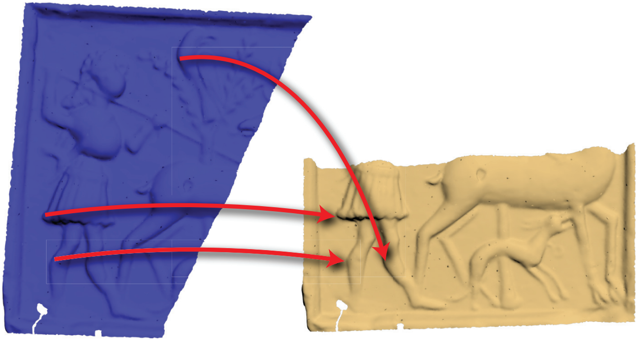


Seeking the density peak

- Density function:

$$\rho(T) = \sum_i K(d(T_i, T))$$

- $T_i \in candidates$
- $d(T_1, T_2)$ - distance function
- $K(r) = e^{-(Dr)^2}$ - Gaussian kernel
- D - spread parameter

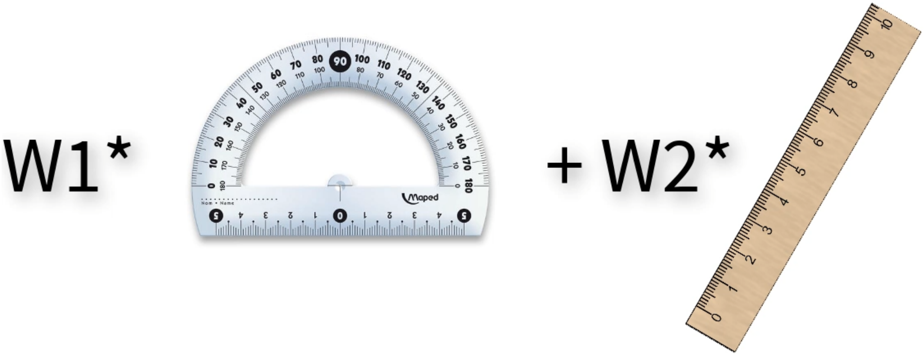
- We do not search for the global maximum of $\rho(T)$
- Only maximum among candidates

Seeking the density peak

- Only small values of $d(T_i, T)$ contribute significantly to $\rho(T)$
- Only T_i for which $d(T_i, T) < r_{max}$ are considered
- Vantage Point Tree [Yia93]
  - Partitions the space based on distances
  - Average query complexity: $\mathcal{O}(log(n))$
  - $d(T_1, T_2)$ must be a **metric**

Transformation Distance Metrics

- Decompose T into rotation R and translation $\mathbf{t}$
- Weighted sum of a rotation metric and the difference of translations



= **composed** metric

Transformation Distance Metrics

Composed metrics

$$d(T_1, T_2) = c_R \frac{d_R(R_1, R_2)}{k_R} + c_t \frac{\|\mathbf{t_1} - \mathbf{t_2}\|}{k_t}$$

- R_1, R_2 - rotation matrices, $\mathbf{t_1}$, $\mathbf{t_2}$ - translation vectors
- C_R, C_t - set the ratio and overall scale
- k_R - normalizes d_R, so that $d_R / k_R \in [0, 1]$
- k_t - normalizes $\|\mathbf{t_1} - \mathbf{t_2}\|$ w.r.t. size of Q

Transformation Distance Metrics

Composed metrics - drawbacks

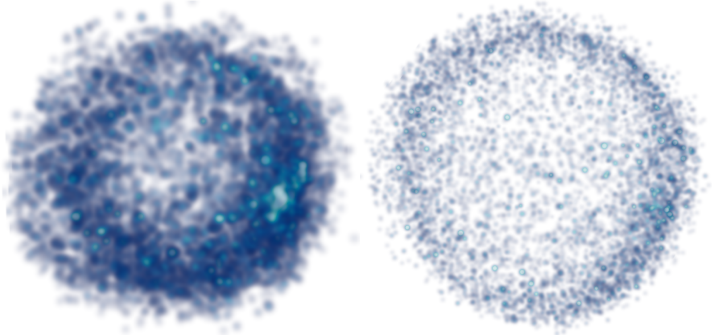
- Order of operations - two equivalent forms of rigid transformations
  - $T(\mathbf{x}) = R\mathbf{x} + \mathbf{t}$ - rotation-first form
  - $T(\mathbf{x}) = R'(\mathbf{x} + \mathbf{t}')$ - translation-first form
  - Different metric values
  - Arbitrary choice
  - We use rotation-first

Transformation Distance Metrics

Composed metrics - drawbacks

- Dependence on position
  - Not depending on position of P
  - Strongly depending on the distance of Q from origin

Centered Q



Non-centered Q

Composed metrics - drawbacks

- Dependence on orientation
  - Independent of orientation of P and Q only if d_R is **bi-invariant**
  - Bi-invariance: $d_R(R_1, R_2) = d_R(R_1 R_0, R_2 R_0) = d_R(R_0 R_1, R_0 R_2)$ for any R_0, R_1, R_2

Transformation Distance Metrics

Composed metrics - Used rotation metrics

1. DEA: $d_R^{DEA}(R_1, R_2) = \sqrt{d(\alpha_1, \alpha_2)^2 + d(\beta_1, \beta_2)^2 + d(\gamma_1, \gamma_2)^2}$
   $d(a, b) = min\{|a - b|, 2\pi - |a - b|\}$
2. DQ: $d_R^{DQ}(R_1, R_2) = min\{\|\mathbf{q}_1 - \mathbf{q}_2\|, \|\mathbf{q}_1 + \mathbf{q}_2\|\}$
3. ADPQ: $d_R^{ADPQ}(R_1, R_2) = arccos(|\mathbf{q}_1^T \mathbf{q}_2|)$
4. DPQ: $d_R^{DPQ}(R_1, R_2) = 1 - |\mathbf{q}_1^T \mathbf{q}_2|$
5. DIM: $d_R^{DIM}(R_1, R_2) = \|I - R_1 R_2^T\|_F = \|R_1 - R_2\|_F$

- $\mathbf{q}$ - unit quaternion corresponding to R
- α, β, γ - Euler angles corresponding to R
- All except DEA are bi-invariant [Huy09]

Transformation Distance Metrics

Composed metrics - Dealing with the drawbacks

- Center Q around the origin, only use bi-invariant rotation metrics

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OR

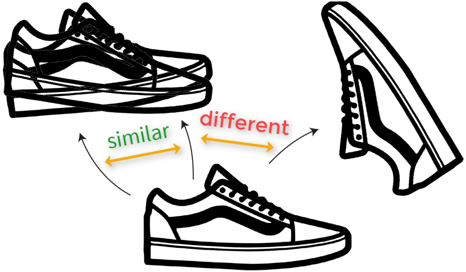
Transformation Distance Metrics

Composed metrics - Dealing with the drawbacks

- Center Q around the origin, only use bi-invariant rotation metrics

OR

- Measure the difference of the **effects** on the data



= **compound** metric

Compound metrics - Vertex Sum of Squares (VSS)

- Squared distances of points transformed by T_1 from points transformed by T_2 [PHYH06]

$$d(T_1, T_2)^2 = \frac{c^2}{k^2} \sum_{i=1}^{\nu_q} \| T_1(\mathbf{q}_i) - T_2(\mathbf{q}_i) \|^2$$

- Only requires one parameter
- c - the scale parameter
- k - normalizes the metric w.r.t. the size and point count of Q
- Independent of position and orientation
- **Can be computed in $\mathcal{O}(1)$**
- Drawback - depends on sampling density

Transformation Distance Metrics

Compound metrics - Triangle Sum of Squares (TSS)

- Solution - integrate over triangles

$$d(T_1, T_2)^2 = \frac{c^2}{k^2} \sum_{i=1}^{\mathcal{T}q} \int_{t_i} \| T_1(\mathbf{x}) - T_2(\mathbf{x}) \|^2 da$$

- t_i - i-th triangle
- c - the scale parameter
- k - normalizes the metric w.r.t. the size and surface area of Q

- Can be expressed in closed form
- **Still $\mathcal{O}(1)$ (precisely)**

Transformation Distance Metrics

Compound metric

Composed metric

Results

- 14 datasets (registration problems)
- Known correct transformations
- Error evaluation:

$$error(T) = \frac{1}{k} \sum_{i=1}^{\mathcal{T}q} \int_{t_i} \| T(\mathbf{x}) - T_c(\mathbf{x}) \| da$$

- T_c - correct transformation
- k - normalizes the error w.r.t. the size and surface area of Q
- The triangle integrals are approximated
- Failure if $error(T) > 0.15$

Results

- 7 different metrics
  - Composed metric with all 5 rotation metrics
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  - Exponential distribution was used
  - $c_R = 1.5 \cdot 2^{i/3}, c_t = 1.5 \cdot 2^{j/3}; i, j = 0, 1, 2, ..., 29$
  - For compound metrics $c = c_R$

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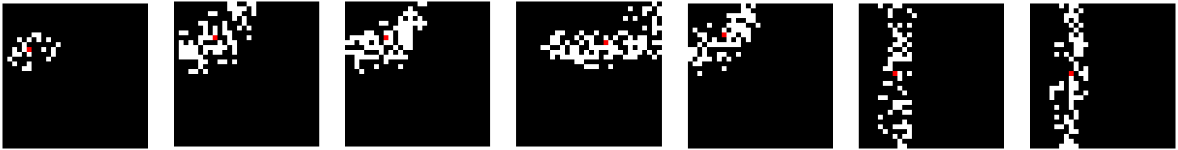
  - For compound metrics $c = c_R$
  - White: good, Black: bad, Red: estimated optimal



DEA DQ ADPQ DPQ DIM VSS TSS

Results

Dataset	DEA		DQ		ADPQ		DPQ		DIM		VSS		TSS	
	Error	FC	Error	FC	Error	FC	Error	FC	Error	FC	Error	FC	Error	FC
Armadillo	0.0683	23	0.0479	0	0.0485	0	0.0486	0	0.0463	0	**0.0455**	0	0.0480	0
Bird	0.0673	0	0.0672	0	0.0672	0	**0.0669**	0	0.0672	0	0.0673	0	0.0672	0
Bubba	0.0032	0	0.0035	0	0.0039	0	0.0059	0	0.0039	0	0.0034	0	**0.0023**	0
Buddha	0.0363	0	0.0255	0	0.0252	0	0.0246	0	**0.0241**	0	0.0242	0	0.0242	0
Coati	0.0283	0	0.0211	0	0.0208	0	0.0212	0	0.0202	0	**0.0199**	0	0.0200	0
Dragon	0.0258	0	0.0222	0	0.0221	0	0.0233	0	0.0221	0	0.0211	0	**0.0209**	0
Eggs	**0.0758**	98	0.0852	120	0.0868	123	0.0967	142	0.1003	143	0.1202	179	0.0915	140
Head	0.0079	0	0.0081	0	0.0082	0	0.0089	0	0.0083	0	0.0081	0	**0.0075**	0
Hippo	0.0760	32	0.0560	0	0.0558	0	0.0566	0	0.0556	0	**0.0528**	0	0.0538	0
Kachel	0.0187	0	**0.0152**	0	0.0156	0	0.0168	0	0.0153	0	0.0156	0	0.0155	0
Oscar	0.0044	0	0.0039	0	0.0040	0	0.0048	0	0.0039	0	0.0036	0	**0.0034**	0
Suzanne	0.0139	0	0.0133	0	0.0139	0	0.0144	0	**0.0130**	0	0.0132	0	**0.0130**	0
Teeth	0.0160	0	0.0137	0	0.0135	0	0.0148	0	0.0131	0	0.0129	0	**0.0127**	0
Testbody	0.0189	0	0.0175	0	0.0172	0	0.0197	0	0.0181	0	0.0186	0	**0.0171**	0
Total	0.0329	153	0.0286	120	0.0288	123	0.0302	142	0.0294	143	0.0305	179	**0.0284**	140

Results of LCP

- LCP used instead of the density peak estimation, fail count measured
- δ' is relative to the size of Q

Dataset	δ' 0.0025	0.005	0.01	0.02	0.04
Arm	619	8	0	0	0
Bir	887	757	633	983	1000
Bub	0	0	0	16	1000
Bud	112	0	0	0	0
Coa	59	0	0	0	0
Dra	330	1	0	0	0
Egg	215	4	0	0	0
Hea	0	0	0	0	645
Hip	708	324	219	76	3
Kac	368	0	0	0	327
Osc	0	0	0	0	0
Suz	90	8	6	438	1000
Tee	0	0	0	0	0
Tes	375	55	1	0	0
Total	3763	1157	859	1513	3975

Conclusion

- Consensus evaluation by **density peak estimation**
  - Can be made **efficient** using Vantage Point Tree
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- It is more **reliable** and **stable** than LCP
- Different transformation distance metrics were compared
  - All seem to be usable
  - Composed metrics have fundamental drawbacks
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  - Composed metrics have fundamental drawbacks
  - Compound metrics have no drawbacks and perform as well or even better
    => There is no reason to prefer composed over compound metrics
  - The proposed TSS metric performs best
- Other possible applications
  - Comparing transformations **only** makes sense when related to input data
  - Compound metrics do that in a systematic way

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Thank you

Feel free to download the reference implementation
`http://meshcompression.org/sgp2019`