Fast Screen Space Curvature Estimation on GPU

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Abstract: Curvature is an important geometric property in computer graphics that provides information about the behavior of object surfaces. The exact curvature can only be calculated for a limited set of surfaces description. Most of the time, we deal with triangles, point sets or some other discrete representation of the surface. For those, curvature computation is problematic. Moreover, most of existing algorithms were developed for static geometry and can be slow for interactive modeling. This paper proposes a screen space method which estimates the mean and Gaussian curvature at interactive rates. The algorithm uses positions and normals to estimate the curvature from the second fundamental form matrix. Using the screen space has advantages over the classical approach: low-poly geometry can be used and additional detail can be added with normal and bump maps. The screen space curvature can be easily added to existing rendering pipelines. The proposed algorithm was tested on several models and it outperforms current state-of-the-art GPU approaches.

1 INTRODUCTION

Visualization of curvature plays an important role in computer graphics. It can help to better understand properties of surfaces and their convex and concave areas. In computer graphics, the basic representation of geometry is a triangle mesh. It represents only an approximation of the original geometry and the same triangle mesh can be obtained for different geometries. Therefore it is only possible to compute an estimation of the curvature of the original smooth surface. For triangle meshes, curvature equals to zero inside triangles, zero or infinity on edges and infinity in vertices. To estimate curvature for a triangle mesh, various algorithms were proposed, using triangle neighbors and approximations to perform the estimation. The more triangles a model has, the better approximation can we get, but the calculation becomes slower. The exact curvature computation cannot be done for volumetric data sets, height fields, point clouds and other discrete representations either.

The curvature approximation can be computationally expensive, especially if the input data are of high quality (many triangles, high volume resolution, large point clouds etc.). A recalculation at each frame during interactive data changes can substantially slow the processing down. Existing methods are mostly used for static geometry and their real-time variants mostly rely on parallelization using GPU.

In the proposed approach, to mitigate this problem, the curvature is not estimated directly from the mesh, but rather from the final rendered image in screen space. Only the currently visible data interesting for the viewer are processed. Calculations are independent of the triangle count of the original geometry, the only limitation is the screen resolution. The advantage is that the curvature can be calculated from any possible model representation with the same algorithm. There is no limitation to triangle meshes, the final scene can contain volumetric models, implicit surfaces, procedurally generated geometry and other screen space generated effects, such as a water surface.

Screen space techniques can be easily added as post-process methods or replace an existing rendering output. Nowadays, these methods are quite popular for many problems, such as water rendering, lighting, ambient occlusion and reflections. In screen space, however, some problems may occur, usually on object edges, where pixel flickering may appear. Another disadvantage stems directly from the screen space itself, where the geometry outside the visible area cannot contribute to the results.

The proposed screen space algorithm is designed to be used as the first and fast estimation of the curvature. For a more precise solution, the curvature should be approximated directly from the underlying models, where connectivity of the triangles can be used to im-

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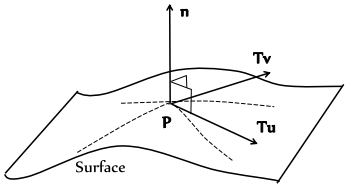


Figure 1: Tangent vectors of surface at point P.

prove the results quality.

Contributions of the proposed solution are:

- it overcomes problems with the interactivity for a high number of triangles by approximating the curvature directly in screen space,
- it can be used in screen space as well as in object space with little or no modification,
- it fits directly into existing rendering pipelines and uses only the usual outputs from deferred renderers (positions, normals).

The rest of this paper is organized as follows. Section 2 covers related work. Section 3 explains the proposed solution in object and screen space. Section 4 presents the algorithm results. Section 5 concludes the paper. Notation used in the article is as follows: symbol "$\cdot$" denotes the dot product, "$\times$" denotes the cross product, $|x|$ is the vector length and $det(X)$ is the determinant of a matrix X.

2 BACKGROUND AND RELATED WORK

2.1 Basic Theory

Only a brief introduction to curvature will be presented. For more details and proofs of theorems, we refer the reader to (Gray, 1997).

The important surface descriptors are the fundamental forms. They describe the first and second order derivatives of a parameterization of the surface at a given point of the surface.

The first fundamental form (I) is constructed from the first order derivatives at a surface point, which give us two tangent vectors (T_u, T_v), see Figure 1. Vectors T_u, T_v are in general not orthogonal. They are, however, orthogonal to the normal vector n to the surface at the given point. Elements of the matrix I are computed as

$$\mathrm{I}=\begin{bmatrix} E & F \\ F & G \end{bmatrix}, \qquad E = T_u \cdot T_u,\ F = T_u \cdot T_v,\ G = T_v \cdot T_v. \tag{1}$$

The second fundamental form (II) is calculated from the second partial derivatives (T_{uu}, T_{vv}, T_{uv}) and the normal vector (n). The elements of the matrix II are computed as

$$\mathrm{II} = \begin{bmatrix} L & M \\ M & N \end{bmatrix}, \qquad n = \frac{T_u \times T_v}{|T_u \times T_v|}, \qquad L = T_{uu} \cdot n,\ M = T_{uv} \cdot n,\ N = T_{vv} \cdot n. \tag{2}$$

Combining the fundamental forms gives the shape operator W (also known as the Weingarten operator). For every point of the surface, it tells us the change of the normalized normal vector in the direction of a tangent vector at this point. W is a 2×2 symmetric matrix that can be obtained from the first (I) and second (II) fundamental forms:

$$W = \mathrm{I}^{-1}\mathrm{II}. \tag{3}$$

The matrix W has two real eigenvalues that correspond to the first (λ_1) and second (λ_2) principal curvature, and its eigenvectors correspond to the principal curvature directions. Mean (H) and Gaussian (K) curvature are computed from principal curvatures as:

$$H = \tfrac{1}{2}(\lambda_1 + \lambda_2), \qquad K = \lambda_1 \lambda_2. \tag{4}$$

2.2 Related Work

As already mentioned, curvature cannot be exactly computed for discrete geometry. Instead of that, only an approximation (or estimation) can be calculated. There are two main categories of approaches - discrete and surface fitting. Discrete methods calculate curvature directly from the data, while surface fitting tries to find a local approximation of the surface and calculates the curvature of this approximation. Usually, discrete methods are faster but less accurate.

There are many algorithms for the curvature estimation, often varying only in details. A comparison can be found in (Magid et al., 2007). Algorithms related to the design of the proposed algorithm are summarized in the next subsections.

2.2.1 Discrete Methods

The discrete method from (Rusinkiewicz, 2004) uses the second fundamental form to calculate the curvature estimate per triangle. These curvatures are then distributed to triangle vertices. Uniform or Voronoi area weights can be used to express the vertex curvature. This step is similar to normal vector calculation for a triangle mesh. The algorithm (Griffin

et al., 2012) is used for volumetric datasets. This algorithm is a variation of (Rusinkiewicz, 2004), running entirely on GPU, and it is optimized for deformable meshes. The interactive nature of this algorithm is achieved by a GPU parallelization rather than algorithmically.

An algorithm similar to (Rusinkiewicz, 2004) has been presented in (Theisel et al., 2004). The triangle curvature computation is based on triangle vertices positions and unnormalized normals. By a linear interpolation, a single point and a normal is calculated for each triangle. The estimated curvature depends on the length and quality of the normals.

The algorithm to compute curvature estimation directly from the triangles can be found in (Meyer et al., 2003). The mean curvature is computed using a discretization of the Laplace operator. Voronoi areas of triangles shared by a vertex are used as weight functions. The Gaussian curvature is calculated from the sum of vertex incident angles, weighted by same Voronoi areas as for the mean curvature.

For a regular height field, curvature can be calculated directly by using Monge Patch (Gray, 1997). From a regular height field, derivatives can be estimated using neighboring points values, which are then used in a series of equations.

2.2.2 Surface Fitting Methods

Surface fitting methods approximate the surface by the least squares techniques. An algorithm for point clouds based on this approach was presented in (Yang and Qian, 2007).

Surface fitting can also be done by finding a local surface approximation, often using a Bézier patch, see (Razdan and Bae, 2005). The curvature is computed at a single vertex directly from the patch approximation. The computational cost is very low, but if the selected neighborhood is small, the results can be incorrect.

Another approach based on Bézier patch was presented in (Zhihong et al., 2011). First, a local Bézier surface is calculated for every triangle. From this surface, curvature can be directly calculated. The Bézier surface patch, however, has no G^1 continuity between neighboring surfaces. On the edges, there could be a steep change in the curvature. An improvement has been proposed by (Fünfzig et al., 2008), where G^1 continuous patch is computed as a blend of Bézier surfaces over neighboring triangles. From this patch, curvature can be directly calculated using an analytic solution as proposed in (Boschiroli et al., 2012). A drawback of this method is that the second derivatives are much more complex than for a simple Bézier patch.

Curvature estimations based on surface fitting in screen space are not very common. The only algorithm known to us is by (Mellado et al., 2013). They propose the screen space algorithm using sphere fitting. For each pixel, the best fitting sphere is found. With this approach, however, only the mean curvature is calculated, while the Gaussian and principal curvatures cannot be computed this way.

3 THE PROPOSED ALGORITHM

The proposed algorithm works in the screen space and it can also be used for classic triangle meshes. The core of the algorithm is similar to (Rusinkiewicz, 2004).

First, a description of the proposed algorithm for a triangle mesh is presented. The screen space version is discussed next.

3.1 BASIC ALGORITHM

The main idea is to describe every triangle independently by the shape operator W(recall Equation (3)). Elements of the shape operator must be calculated to find eigenvalues of the matrix and the final curvatures.

The proposed method uses a local orthonormal basis. In such a case, the first fundamental form (I) becomes an identity matrix, which means that the second fundamental form (II) is equivalent to the shape operator, i.e. $W = \mathrm{II}$.

To eliminate one dimension, every triangle is transformed to a local coordinate system, also known as the tangent space. Once the triangle is in the local space, one of the dimensions is constant and represents the plane of the triangle. In the following calculations, this dimension is not used and the problem is reduced from 3D to 2D.

3.1.1 Triangle Conversion to the Local System

For every triangle, given by its vertices V_1, V_2, V_3 and normal vectors Vn_1, Vn_2, Vn_3, a local orthonormal coordinate system is built (see Figure 2).

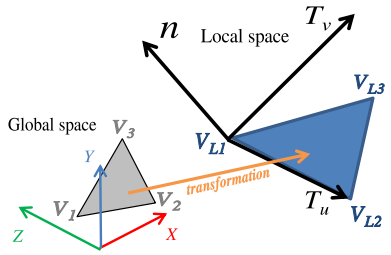


Figure 2: Local triangle transformation.

One of the triangle vertices is selected as the basis origin and subtracted from every triangle vertex. The matrix which transforms the triangle to the local system is created from the three-dimensional basis vectors T_u, T_v, n. These vectors are obtained using

$$T_u = \frac{V_2 - V_1}{|V_2 - V_1|},$$
$$n = \frac{T_u \times (V_3 - V_1)}{|T_u \times (V_3 - V_1)|}, \quad (5)$$
$$T_v = \frac{T_u \times n}{|T_u \times n|}.$$

The original triangle is expressed in this local system, resulting in vertices V_{L1}, V_{L2}, V_{L3} and normals $Vn_{L1}, Vn_{L2}, Vn_{L3}$. Sample conversion of one of the triangle vertices from the global to the local system is calculated as:

$$V_{L2} = \begin{bmatrix} T_{ux} & T_{uy} & T_{uz} \\ T_{vx} & T_{vy} & T_{vz} \\ n_x & n_y & n_z \end{bmatrix} (V_2 - V_1) \ . \quad (6)$$

Normals should be converted using the inverse transposed matrix. Due to the orthonormality of the system, the matrix inverse is not needed, since an inversion of an orthonormal matrix is equal to matrix transposition.

3.1.2 The Curvature Calculation

The triangle in the local space is used to build the shape operator (see in Equation (2)), where variables L, M, N are unknown.

The shape operator describes the change of the normal along an edge of the triangle. The triangle is in the local space and one of the coordinates (normal) is constant. This coordinate is left out, which leads to 2D vectors instead of 3D. The edges of the triangle are expressed as 2D vectors

$$(u_i, v_i)^T = V_{Li} - V_{L(i+1) \bmod 3}, \quad (7)$$

and changes of the triangle normals are 2D vectors

$$(dNu_i, dNv_i)^T = Vn_{Li} - Vn_{L(i+1) \bmod 3}, \quad (8)$$

where $i = 1, 2, 3$ is the triangle edge index.

Changes of normals along the edges of the triangle are known. These changes together with edge direction vectors are used to create a system of equations to find the unknown variables L, M, N. For one edge of the triangle, we get an underdetermined system

$$\begin{bmatrix} L & M \\ M & N \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} dNu_1 \\ dNv_1 \end{bmatrix}. \quad (9)$$

However, by constructing the same system for every edge of the local space triangle, an overdetermined system is obtained. The system is in the form $Ax = b$, the least squares method is used to obtain unknown variables:

$$x = \ (A^T A)^{-1} A^T b \ . \quad (10)$$

In this particular case, the matrix A is built from the triangle edge vectors $(u_i, v_i)^T, i = 1, 2, 3$ and b is the vector of changes of the triangle normals $(dNu_i, dNv_i)^T, i = 1, 2, 3$. Index i denotes the triangle edge index. The final matrices are as follows:

$$A = \begin{bmatrix} u_1 & v_1 & 0 \\ 0 & u_1 & v_1 \\ u_2 & v_2 & 0 \\ 0 & u_2 & v_2 \\ u_3 & v_3 & 0 \\ 0 & u_3 & v_3 \end{bmatrix}, b = \begin{bmatrix} dNu_1 \\ dNv_1 \\ dNu_2 \\ dNv_2 \\ dNu_3 \\ dNv_3 \end{bmatrix}, x = \begin{bmatrix} L \\ M \\ N \end{bmatrix}. \quad (11)$$

The total number of numerical operations can be decreased by optimization. First, a substitution $B = A^T A$ is introduced. The matrix B is symmetric and its elements can be represented by variables p, q, r:

$$B = A^T A = \begin{bmatrix} p & q & 0 \\ q & p+r & q \\ 0 & q & r \end{bmatrix},$$
$$p = u_1^2 + u_2^2 + u_3^2, \quad (12)$$
$$q = u_1 v_1 + u_2 v_2 + u_3 v_3,$$
$$r = v_1^2 + v_2^2 + v_3^2.$$

The inverse of the matrix B can be computed using Equation (13). Since B is symmetric, the computation is fast and easy.

$$B^{-1} = det(B) \begin{bmatrix} p(r+p) - q^2 & -qr & q^2 \\ -qr & pr & -pq \\ q^2 & -pq & p(r+p) - q^2 \end{bmatrix} \quad (13)$$

The final step is to calculate values for the unknown vector x. A part of this step can be simplified, because the inverse of the matrix B is symmetric (see Equation (14)) and the matrix A has many zero elements. A simplified multiplication is expressed by Equation (15).

$$B^{-1} = det(B) \begin{bmatrix} b_1 & b_2 & b_3 \\ b_2 & b_4 & b_5 \\ b_3 & b_5 & b_6 \end{bmatrix} \quad (14)$$

$$B^{-1} A^T = det(B) *$$
$$\left(\begin{bmatrix} u_1 b_1 & u_1 b_2 & u_2 b_1 & u_2 b_2 & u_3 b_1 & u_3 b_2 \\ u_1 b_2 & u_1 b_4 & u_2 b_2 & u_2 b_4 & u_3 b_2 & u_3 b_4 \\ u_1 b_3 & u_1 b_5 & u_2 b_3 & u_2 b_5 & u_3 b_3 & u_3 b_5 \end{bmatrix} + \right.$$
$$\left. \begin{bmatrix} v_1 b_2 & v_1 b_3 & v_2 b_2 & v_2 b_3 & v_3 b_2 & v_3 b_3 \\ v_1 b_4 & v_1 b_5 & v_2 b_4 & v_2 b_5 & v_3 b_4 & v_3 b_5 \\ v_1 b_5 & v_1 b_6 & v_2 b_5 & v_2 b_6 & v_3 b_5 & v_3 b_6 \end{bmatrix} \right) \quad (15)$$

Having obtained the final vector x, we can construct the desired shape operator. From this matrix, the eigenvalues λ_1, λ_2 are computed by solving the characteristic polynomial. These values correspond to the principal curvature estimated for the triangle. These curvatures can be used to evaluate the mean and Gaussian curvature (see Equation (4)).

The presented algorithm computes the curvature for each triangle. To obtain the curvature at the vertices, we have to use all adjacent triangles at the given point. The final curvature can be estimated as an average from all adjacent triangles or the curvature can be further weighted by the triangle area.

In the above calculations, an overdetermined system was constructed from all three edges of the triangle. To solve the system, only two edges are sufficient (values for $i = 3$ will be zero). Differences in both approaches are discussed in Section 4.

3.2 Screen Space Version

The screen space version of the proposed algorithm fits directly into an existing deferred rendering pipeline. Only normal and depth (from which the position is reconstructed) is required for every pixel. There could be probably some quality improvements, if additional information (id of the triangle to which the current pixel belongs, the triangle size in the screen space etc.) were available, but this is not the current target.

The screen space depth buffer can be interpreted as a 2.5D function with an underlying regular grid and function values of the depth. In the screen space, there is a constant step size between neighboring pixels. Those pixels are triangulated and each pixel center is taken as a triangle vertex. One possible local triangulation can be seen in Figure 3. This screen space triangulation is converted to the world or camera space by reconstruction of the position and the normal for each pixel. This creates a simple triangulated mesh and the curvature is estimated on this mesh using the technique described in Section 3.1.

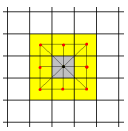


Figure 3: 1-ring neighborhood pixels.

The algorithm from Section 3.1 can be used directly in the screen space. It can run entirely on the GPU, using a pixel shader. The inverse matrix can be computed very fast (Equation (13)) and only six values have to be stored due to the matrix symmetry.

If all three edges of each triangle are used, there is a limitation in Equation (15) caused by shaders, where maximal dimension of the native data type can be four, but 3×6 matrix and 6×1 vector are needed. If the simplified matrices from Equation (15) are used, the calculation can be split into two parts. Each of these parts has a halved dimension (Equation (16)) of the original matrix.

$$B_1 = \begin{bmatrix} u_1b_1 + v_1b_2 & u_1b_2 + v_1b_3 & u_2b_1 + v_2b_2 \\ u_1b_2 + v_1b_4 & u_1b_4 + v_1b_5 & u_2b_2 + v_2b_4 \\ u_1b_3 + v_1b_5 & u_1b_5 + v_1b_6 & u_2g + v_2b_5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} u_2b_2 + v_2b_3 & u_3b_1 + v_3b_2 & u_3b_2 + v_3b_3 \\ u_2b_4 + v_2b_5 & u_3b_2 + v_3b_4 & u_3b_4 + v_3b_5 \\ u_2b_5 + v_2b_6 & u_3b_3 + v_3b_5 & u_3b_5 + v_3b_6 \end{bmatrix},$$

$$x = det(B) \left(B_1 \begin{bmatrix} dNu_1 \\ dNv_1 \\ dNu_2 \end{bmatrix} + B_2 \begin{bmatrix} dNv_2 \\ dNu_3 \\ dNv_3 \end{bmatrix} \right) \tag{16}$$

If only two edges are used, calculations can be computed even more efficiently on the GPU:

$$x = det(B) \begin{bmatrix} u_1b_1 + v_1b_2 & u_1b_2 + v_1b_3 \\ u_1b_2 + v_1b_4 & u_1b_4 + v_1b_5 \\ u_1b_3 + v_1b_5 & u_1b_5 + v_1b_6 \end{bmatrix}$$

$$\begin{bmatrix} u_2b_1 + v_2b_2 & u_2b_2 + v_2b_3 \\ u_2b_2 + v_2b_4 & u_2b_4 + v_2b_5 \\ u_2g + v_2b_5 & u_2b_5 + v_2b_6 \end{bmatrix} \begin{bmatrix} dNu_1 \\ dNv_1 \\ dNu_2 \\ dNv_2 \end{bmatrix}. \tag{17}$$

All calculations are based on triangles that need to be reconstructed in the screen space. They are obtained directly from the currently rendered pixel and its neighbors, see again Figure 3. However, if the neighborhood width is only one pixel (as in Figure 3), according to our tests, a single triangle suffices to compute the curvature estimation.

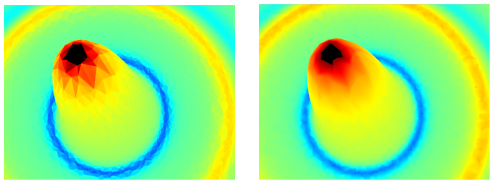
3.3 Level of Detail

In the screen space, visible details often depend on the camera distance from the scene. Small triangles in the world space may occupy almost all the pixels of the rendered image if the camera is very close to the surface. If the camera is far away, the same triangle can take only one pixel of the final image. Considering this, a level of detail technique can be used to improve the visual quality of the estimated curvature.

If only a basic 1-ring neighborhood is used (see Figure 3), triangles of the original mesh can be seen in the estimated curvature (see Figure 4(a)). The estimated curvature within every triangle is constant. GPU interpolates normals and positions during rendering, leading to smooth Phong shading, but the proposed method uses differences in the positions and

normals which are constant (except for the numerical errors) for the flat geometry. It leads to constant curvature at every pixel of each triangle.

To mitigate this problem, level of detail (LOD) sampling can be used. For closer points, triangles are constructed from a wider neighborhood. Using this approach, the curvature should be computed from more than one triangle. A maximal number of eight triangles per pixel, creating a triangle fan, is sufficient according to our experiments. The final curvature is calculated as an average value from all triangles. The result with LOD is shown in Figure 4(b).



(a) Without LOD (b) With LOD

Figure 4: Screen space curvature.

3.4 Limitations

Similarly to other screen space techniques, the proposed algorithm has its disadvantages. When two neighboring pixels do not come from the same part of the surface, there appears a surface discontinuity between those pixels and an artifact in the computed curvature may appear.

If only 1-ring neighborhood is used, this problem is not significant and can be sometimes ignored. It causes incorrect curvature on edges, but those incorrect values are only one pixel wide.

Incorrect values are more problematic when LOD is used. The reconstructed triangles may consist of points, which come from a discontinuous surface. This situation cannot be distinguished directly in the screen space. To solve this problem, conditions based on the depth can be used. If the depth value of a screen space triangle is bigger than a threshold, the triangle is rejected and not used in further computations. If all triangles are rejected, we use only one triangle without LOD to estimate the curvature for the current pixel. This approach is not optimal and in the future, we would like to improve it in the future by using a different triangle reconstruction technique or by detecting problematic cases more accurately than by a threshold.

4 EXPERIMENTS AND RESULTS

To test the proposed method, a PC with the following configuration was used: Intel Core i7 CPU running at 4GHz, 32GB of RAM memory, nVidia Geforce 960GTX graphics card with 2GB of video memory. The algorithm was implemented in C++ and OpenGL 4.4 with GLSL shaders.

The implementation of the algorithm by (Mellado, 2015), based on (Mellado et al., 2013), was done using GLSL instead of CUDA used in the original paper.

The color gradient used for all visualisations goes from a blue for negative values to a red color for positive values. A green color in the middle represents zero. See Figure 5.

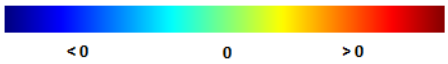


Figure 5: Color gradient used in presented visualizations.

4.1 Curvature Error

In this section, comparison of the proposed method for a triangle mesh, as defined in Section 3.1, and exactly computed curvature from analytic surfaces is provided. Every test uses exact unit-length normals computed from the function itself. In the comparisons, two and three edges were used to create the overdetermined system of equations.

The proposed method on the triangle mesh has been also tested against the Bézier triangles algorithm from (Zhihong et al., 2011).

First, a sphere was tested. A sphere has a constant mean and Gaussian curvature, dependent on the sphere radius r. Curvatures on the sphere can be calculated as $H = \frac{1}{r^2}$ and $K = -\frac{1}{r}$. As a discrete representation of the sphere, a subdivided icosahedron with an exact normals and radius 6 was used. The proposed method in both variations has a constant mean squared error (MSE) with value $8.2 * 10^{-16}$ for Gaussian and $7.8 * 10^{-17}$ for mean curvature. For different radii, MSE has a similar behavior.

Next, two analytic functions were tested (see Figure 6). The function f_1 has convex and concave parts, a high peak at its center, and it is undefined at the point $[0,0]$ (at this point, division by zero would occur). Function f_2 has a saddle shape with minor bumps.

For tests, the functions were tessellated using a Delaunay triangulation in the XY plane using uniformly distributed points with $x,y \in [-10,10]$. The MSE value gives the error of the proposed method on the triangle mesh in comparison with the curvature analytically computed from the input function. See Figure 7.

Results of the comparison are in Figure 8. Small peaks in the graph are caused by randomness of the point positions. It is more visible for f_1 due to its

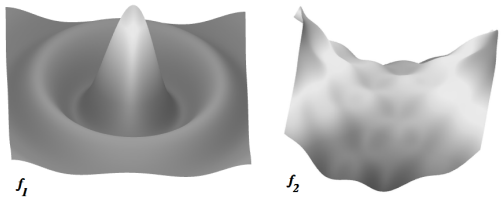
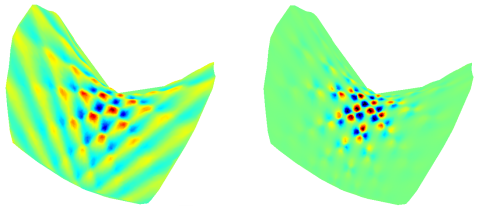


Figure 6: Tested functions $f_1 = 10\frac{sin(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}, (x \neq 0), (y \neq 0)$, $f_2 = sin(x)cos(y) + 0.1(x^2 - y^2)$, $x, y \in < -10; 10 >$.



(a) Mean curvature (b) Gaussian curvature

Figure 7: Curvatures of f_2 calculated from the triangle mesh.

peak around the point $[0,0]$. For denser tesellations, there is a small difference in using two or three edges of the triangle to solve the system. In some cases, two edges offer better results and vice versa.

Another comparison of the proposed method was done against the algorithm (Zhihong et al., 2011) using Bézier triangles. This algorithm was chosen according to the promising results published in the original paper. The algorithm has worse results on the triangle meshes created using random points, see Figure 9. The MSE values were varying from 0.5 to almost

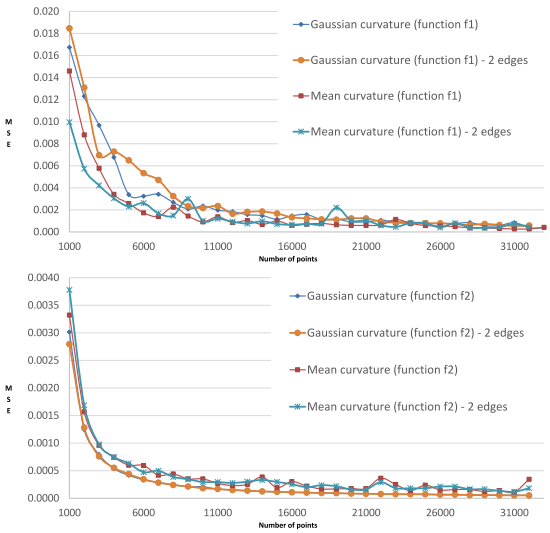
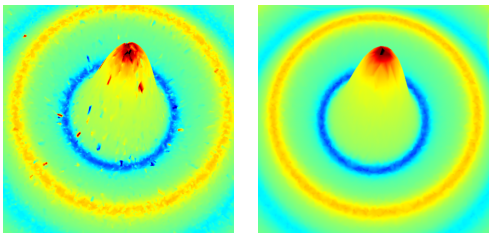


Figure 8: MSE of the method for the triangle mesh compared to the analytically computed curvature evaluated directly from the implicit function.



(a) (Zhihong et al., 2011) (b) Proposed method

Figure 9: The comparison of the curvature of f_1, compared on a tessellation created on random points.

40. For most of the triangles, the calculated curvature gives the error comparable with our proposed method, however, large error values appeared for some small or sliver triangles resulting in too arched Bézier triangles. This problem is not present in the proposed method.

4.2 Screen Space Comparison

The comparison of the screen space method is done against the curvature calculated by the proposed method using three edges directly on the triangle mesh with and without the LOD active. Even with only two edges, the results were almost identical (about 2% difference in calculated values). The proposed algorithm was also compared with (Mellado et al., 2013), the only other screen space technique known to us.

The tested models are: Stanford Dragon (300 000 vertices), MaxPlanck (152 403 vertices), Function f_1 (15 000 vertices) and Subdivided icosahedron (varying number of vertices). In the screen space, the quality of the computed curvature depends on the camera distance from the model. If we compute the curvature for the triangle mesh and render the result, with the camera moving away from the model, the triangles become smaller and more triangles can be rendered in the same pixel. This can cause an incorrect curvature to be visualized. In the proposed screen space method, the problem associated with rasterization cannot happen, because only the visible parts are used to calculate the result and only one value is used for the final pixel. In every test, the model was tested as fully visible on the screen and the camera was moving away from the model. The dependency of MSE on the distance between the viewer and the model is shown in the following graphs.

The test with recursively subdivided icosahedron sphere with radius 6 showed a similar behavior to the per vertex computed values from tests in Section 4.1. The only difference was a small noise between edges of the model and the background of the scene.

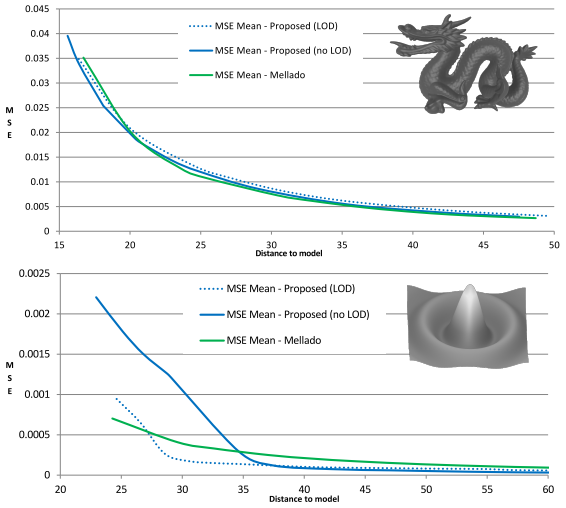


Figure 10: Comparison of the screen space MSE for the mean curvature calculated directly from the triangle mesh. The proposed method with, without LOD and algorithm from (Mellado et al., 2013) (Mellado) were tested.

In all other tests, remaining models were used. From the graphs in Figure 10 it can be seen that the quality of both screen space algorithms is comparable for the mean curvature. For the dragon model, using LOD has a little or no effect at all. The original model has a dense tessellation and LOD can skip fine details. On the other hand, for the model of the function, the proposed method with LOD achieves better quality.

Gaussian curvature comparison was done only with and without LOD, since there is no other screen space method known to us that calculates the Gaussian curvature. See results in Figure 11. The behavior is similar to Figure 10, with a roughly doubled amount of the MSE error. This is caused by the curvature calculation, where the mean curvature is only a sum of the principal ones, while the Gaussian is computed by multiplying principal curvatures. In that case, the errors of both values are multiplied as well.

The visual comparison of the proposed method with (Mellado et al., 2013) can be seen in Figure 12. Both algorithms have a comparable visual quality. The proposed method results look sharper, (Mellado et al., 2013) is more blurry. See Figure 13 for comparison of the quality of the proposed method in the screen space against the same method in the object space.

No LOD is used to show real differences based on the camera distance. For the camera at a greater distance (full model), there is almost no visible difference. With camera closer to the surface (the detailed model), the triangles of the mesh begin to appear in the screen space curvature.

The effect of LOD can be seen in Figures 14 - 16.

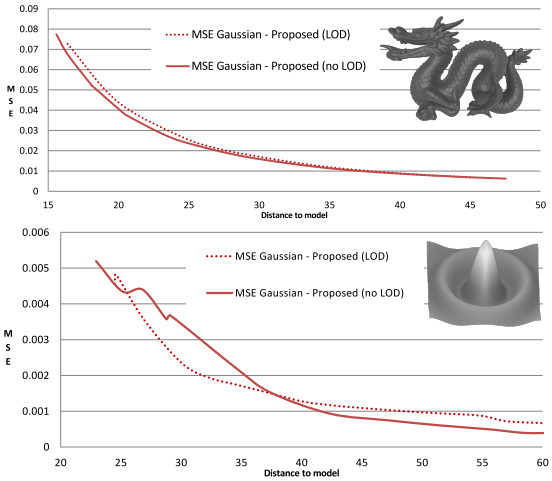
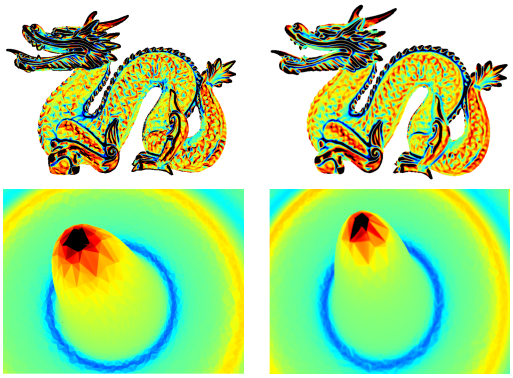


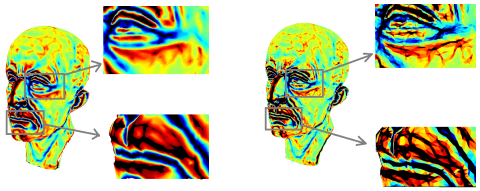
Figure 11: Comparison of the proposed screen space MSE for the Gaussian curvature calculated directly from the triangle mesh.



(a) Proposed method (b) (Mellado et al., 2013)

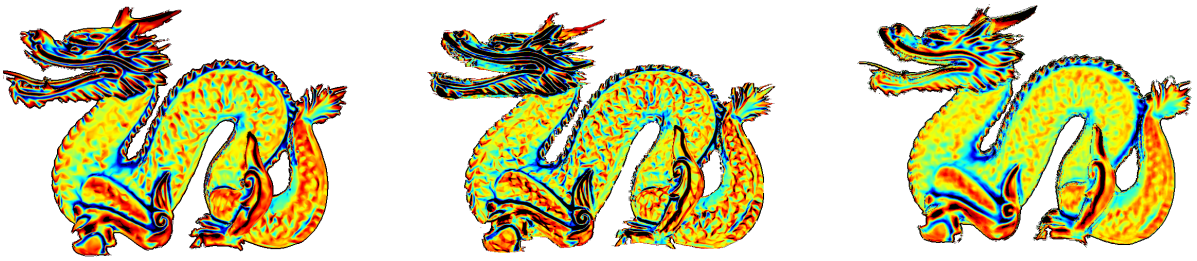
Figure 12: Comparison of the mean curvature. As (Mellado et al., 2013) has no LOD, the presented comparison also uses none.

If the camera is moving away from the mesh, there is a distance, from which further there is a small or no difference between using and not using LOD. In some cases, using LOD can bring worse results as it smooths out fine details (see Figure 14). On the other hand, in the example of the Gaussian curvature in Figure 15, the use of LOD improved the result consider-



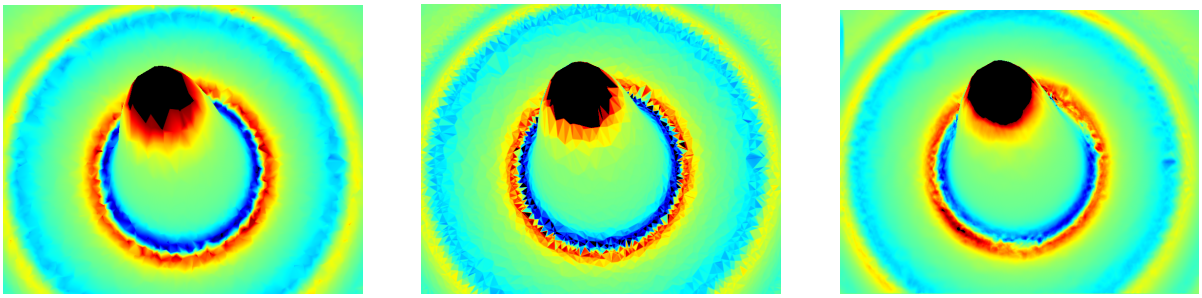
(a) Object space (b) Screen space without LOD

Figure 13: Comparison of the mean curvature for the Max-Planck model.



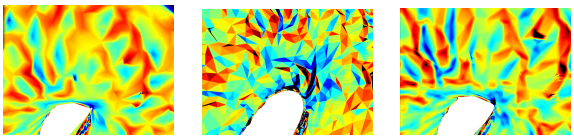
(a) Object space (b) Screen space without LOD (c) Screen space with LOD

Figure 14: Comparison of the mean curvature.



(a) Object space (b) Screen space without LOD (c) Screen space with LOD

Figure 15: Comparison of Gaussian curvature using function f_1 from Section 4.1.



(a) Object space (b) Screen space without LOD (c) Screen space with LOD

Figure 16: Detail of the mean curvature.

ably. Another comparison can be seen in the closeup in Figure 16. If the camera moves very close to the surface, LOD is required to obtain a smooth result. Without LOD, the computed curvature appears as random colors. In some cases, e.g., in wireframe view, this visualization can be sometimes enough to see the shape. To set a suitable distance for LOD is, however, difficult - the same value does not work for all models.

4.3 Performance

The proposed method runs at interactive frame rates. Due to the independence on the geometry, all tested models brought nearly the same results. In the tests, the method was computed for pixel coverage of $2-100$ per cent of the screen. The depth value of the remaining pixels was set to infinity to discard these pixels. The comparison was done against the screen space method from (Mellado et al., 2013).

The resulting performance can be seen in Figures 17 and 18. In both tests, a decrease of performance is partially caused by LOD computation itself but mostly by the need of branches in the pixel shader to decide if the triangle can be used or will be rejected as described in Section 3.3. With a comparable visual quality as (Mellado et al., 2013) (no LOD used), the proposed algorithm is much faster if calculations use only two edges of the triangle.

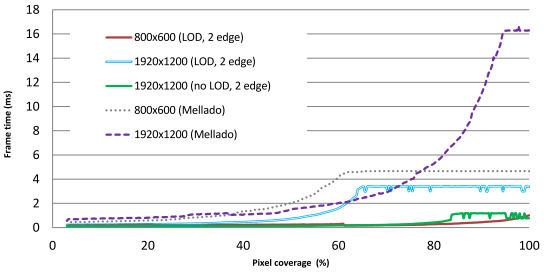


Figure 17: Frame time based on screen pixel coverage. The proposed algorithm in version with only two edges (with and without LOD) against (Mellado et al., 2013) (Mellado) were tested.

In Figure 18, the same scene was tested with all three edges used for computation. This version is considerably slower, due to the solution of Equation (16). In the two edge solution, only one matrix is needed (Equation (17)).

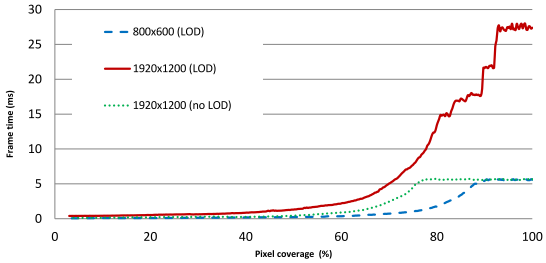


Figure 18: Proposed algorithm tested with all 3 edges of the triangle used for the curvature calculation.

5 CONCLUSIONS

This paper presented an algorithm for estimating curvature in screen space which can compute principal curvatures, is easy to implement and runs in real-time. It can be easily added to existing rendering pipelines. Apart from the screen space, the same algorithm can be used to compute the curvature directly from the triangle meshes.

The limitations of this technique are similar to other screen-space algorithms. There are possible problems with surface discontinuities. Estimated curvature depends on the distance of the mesh from the camera, where small details are smoothed if camera is far away from the surface.

In the future work, further LOD improvement and stability during view changes are planned.

We would also like to use the algorithm for the ambient occlusion estimation. Some research has been done on this topic by (Hattori et al., 2011), but the curvature was precalculated from the model in object space.

A reference implementation of the proposed method (shader source code and test application) is available at http://graphics.zcu.cz/ sscurvature.html.

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